Correction Devoir n° 7 66T

/36 /20

1# 1) $\int \sqrt{x}\,(2x+1)\,dx = (2x+1)\cdot\frac{2}{3}\cdot x^{3/2} - \int 2\cdot\frac{2}{3}\cdot x^{3/2}\,dx$

$f(x)=2x+1$	$f'(x)=2$
$g'(x)=\sqrt{x}$ $=x^{1/2}$	$g(x)=\frac{2}{3}x^{3/2}$

$= (2x+1)\cdot\frac{2}{3}\cdot x^{3/2} - \frac{4}{3}\int x^{3/2}\,dx$

$= \frac{2}{3}(2x+1)\sqrt{x^3} - \frac{4}{3}\,\frac{x^{5/2}}{\frac{5}{2}} + k$

$= \frac{2}{3}(2x+1)\sqrt{x^3} - \frac{8}{15}\sqrt{x^5} + k$

2) $\int \frac{5-x}{(2x+1)^2}\,dx = \int (5-x)\cdot(2x+1)^{-2}\,dx$

$f(x)=5-x$	$f'(x)=-1$
$g'(x)=(2x+1)^{-2}$	$g(x)=-\frac{1}{2(2x+1)}$

$\int (2x+1)^{-2}\,dx = \int t^{-2}\,\frac{dt}{2}$

$t=2x+1$

$dt=2dx$

$\frac{dt}{2}=dx$

$= \frac{1}{2}\cdot\frac{t^{-1}}{-1} + k$

$= -\frac{1}{2(2x+1)} + k$

$\int \frac{1}{2x+1}\,dx = \int \frac{1}{t}\,\frac{dt}{2}$

$t=2x$

$dt=2dx$

$\frac{dt}{2}=dx$

$= \frac{\ln(|t|)}{2}$

$= \frac{\ln(|2x+1|)}{2}$

$= (5-x)\cdot\left(-\frac{1}{2(2x+1)}\right) - \int -1\cdot\left(-\frac{1}{2(2x+1)}\right)dx$

$= \frac{-(5-x)}{2(2x+1)} - \frac{1}{2}\int\frac{1}{2x+1}\,dx$

$= \frac{x-5}{2(2x+1)} - \frac{1}{2}\cdot\frac{\ln(|2x+1|)}{2} + k$

$= \frac{x-5}{2(2x+1)} - \frac{1}{4}\cdot\ln(|2x+1|) + k$

3) $\int x^2 e^{-x}\,dx = -x^2e^{-x} - \int 2x\cdot(-e^{-x})\,dx$

$f(x)=x^2$	$f'(x)=2x$
$g'(x)=e^{-x}$	$g(x)=-e^{-x}$

$f(x)=x$	$f'(x)=1$
$g'(x)=e^{-x}$	$g(x)=-e^{-x}$

$= -x^2e^{-x} + 2\int x e^{-x}\,dx$

$= -x^2e^{-x} + 2\left(x\cdot(-e^{-x}) - \int 1\cdot(-e^{-x})\right)$

$= -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}\,dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + k$

$= e^{-x}(-x^2-2x-2) + k$

ou $= -e^{-x}(x^2+2x+2) + k$

2 # 1) $\int_{-2}^{1}\frac{3x}{x^2+1}\,dx = 3\int_{5}^{2}\frac{1}{t}\,\frac{dt}{2}$

$t=x^2+1$

$dt=2x\,dx$

$\frac{dt}{2}=x\,dx$

$x=-2\to t=(-2)^2+1=5$

$x=1\to t=1^2+1=2$

$= \frac{3}{2}\left[\ln(|t|)\right]_5^2$

$= \frac{3}{2}\left[\ln(2)-\ln(5)\right]$

$\simeq -1{,}374$

2) $\int_{-5}^{-1}\frac{x-3}{\sqrt{x^2-6x}}\,dx = \int_{55}^{7}\frac{1}{\sqrt{t}}\,\frac{dt}{2} = \frac{1}{2}\int_{55}^{7} t^{-1/2}\,dt$

$t=x^2-6x$

$dt=2x-6$

$\frac{dt}{2}=(x-3)\,dx$

$x=-5 \to t=25+30=55$

$x=-1 \to t=1+6=7$

$= \frac{1}{2}\left[\frac{t^{1/2}}{\frac{1}{2}}\right]_{55}^{7}$

$= \sqrt{7}-\sqrt{55} \simeq -4{,}77$

3) $\int_0^{\frac{\pi}{3}} x \cdot \sin(3x)\, dx = \left[-\frac{x \cdot \cos(3x)}{3} \right]_0^{\frac{\pi}{3}}$ /1

$f(x) = x$	$f'(x) = 1$
$g'(x) = \sin(3x)$	$g(x) = -\frac{\cos(3x)}{3}$

/2

$- \int_0^{\frac{\pi}{3}} 1 \cdot \left(-\frac{\cos(3x)}{3} \right) dx$

$= \left[-x \cdot \frac{\cos(3x)}{3} \right]_0^{\frac{\pi}{3}}$ /1

$+ \frac{1}{3} \underbrace{\int_0^{\frac{\pi}{3}} \cos(3x)\, dx}_{\frac{\sin(3x)}{3}}$ /1

/5 $= \left[-x \cdot \frac{\cos(3x)}{3} \right]_0^{\frac{\pi}{3}} + \left[\frac{\sin(3x)}{9} \right]_0^{\frac{\pi}{3}}$

$= -\frac{\pi}{3} \cdot \frac{\cos(\frac{\pi}{3} \cdot 3)}{3} - \left[-0 \cdot \frac{\cos(3 \cdot 0)}{3} \right] + \frac{\sin(3 \cdot \frac{\pi}{3})}{9} - \frac{\sin(3 \cdot 0)}{9}$

$= -\frac{\pi}{9} \cdot \overbrace{\cos(\pi)}^{=-1} - 0 + \underbrace{\frac{\sin(\pi)}{9}}_{=0} - \underbrace{\frac{\sin(0)}{9}}_{=0} = \frac{\pi}{9}$

---

4) $\int_{\ln(2)}^{\ln(5)} \frac{e^x}{(2e^x+3)^2} dx = \int_7^{13} t^{-2} \frac{dt}{2}$ /5

$t = 2e^x + 3$

$\frac{dt}{2} = \frac{\cancel{2} e^x}{\cancel{2}} dx$ /2

$x = \ln(5) \to t = 2e^{\ln(5)} + 3 = 13$

$x = \ln(2) \to t = 2e^{\ln(2)} + 3 = 7$

$= \frac{1}{2} \left[\frac{t^{-1}}{-1} \right]_7^{13}$ /2 /2

$= -\frac{1}{2} \left[\frac{1}{t} \right]_7^{13}$ /1

$= -\frac{1}{2} \left[\frac{1}{13} - \frac{1}{7} \right] = \frac{13-7}{2 \cdot 91} = \frac{6}{2 \cdot 91} = \frac{3}{91}$