Correction Devoir n° 6 : Primitives par substitution. 6 GT

1) $\int \sin(-3x+2)\,dx = \int \sin(t)\,\frac{dt}{-3}$ /1 4

$t = -3x+2$

$dt = -3\,dx$

$\frac{dt}{-3} = dx$ 1,5

$= -\frac{1}{3}\int \sin(t)\,dt = -\frac{1}{3}(-\cos(t)) + k$ /1

$= \frac{1}{3}\cos(-3x+2) + k$ 0,5

2) $\int x\cos(x^2)\,dx = \int \cos(t)\,\frac{dt}{2} = \frac{1}{2}\int \cos(t)\,dt$

$t = x^2$

$dt = 2x\,dx$

$\frac{dt}{2} = x\,dx$

$= \frac{1}{2}\sin(t) + k$

$= \frac{1}{2}\sin(x^2) + k$

3) $\int \frac{1}{\sqrt{-5x+2}}\,dx = \int t^{-1/2}\,\frac{dt}{-5} = -\frac{1}{5}\int t^{-1/2}\,dt$

$t = -5x+2$

$dt = -5\,dx$

$\frac{dt}{-5} = dx$

$= -\frac{1}{5}\,\frac{t^{1/2}}{\frac{1}{2}} + k$

$= -\frac{2}{5}\sqrt{t} + k$

$= -\frac{2}{5}\sqrt{-5x+2} + k$

4) $\int 4\cdot(5x-2)^3\,dx = 4\int t^3\,\frac{dt}{5} = \frac{4}{5}\cdot\frac{t^4}{4} + k$

$t = 5x-2$

$\frac{dt}{5} = \frac{5\,dx}{5}$

$= \frac{(5x-2)^4}{5} + k$

5) et 6) inversés

6) $\int \frac{1+e^{2x}}{2x+e^{2x}}\,dx = \int \frac{1}{t}\,\frac{dt}{2} = \frac{1}{2}\ln(|t|) + k$

voir plus loin pour le "t"

$t = 2x + e^{2x}$

$dt = 2 + 2e^{2x}\,dx$

$\frac{dt}{2} = 1 + e^{2x}\,dx$

$= \frac{1}{2}\ln(|2x + e^{2x}|) + k$

5) $\int \frac{2x}{(7x^2+1)^5}\,dx = 2\int t^{-5}\,\frac{dt}{14}$

$t = 7x^2+1$

$dt = 14x\,dx$

$\frac{dt}{14} = x\,dx$

$= \frac{2}{14}\,\frac{t^{-4}}{-4} + k$

$= -\frac{1}{28\cdot t^4} + k$

$= \frac{-1}{28(7x^2+1)^4} + k$

7) $\int \cos(x)\cdot\sin^2(x)\,dx = \int t^2\,dt$

$t = \sin(x)$

$dt = \cos(x)\,dx$

$= \frac{t^3}{3} + k$

$= \frac{\sin^3(x)}{3} + k$

8) $\int e^{3-x}\,dx = \int e^t(-dt)$

$t = 3-x$

$dt = -dx$

$-dt = dx$

$= -e^t + k = -e^{3-x} + k$

9) $\int \underbrace{\frac{1}{x-2}}_{\text{I}} - \underbrace{\frac{2}{3-x}}_{\text{II}}\,dx = \int \frac{1}{x-2}\,dx - 2\int \frac{1}{3-x}\,dx$

I : $t = x-2$

$dt = dx$

$\int \frac{1}{x-2}\,dx = \int \frac{1}{t}\,dt = \ln(|t|) + k$

$= \ln(|x-2|) + k$ 5

(ii) $t = 3-x$ $\qquad \int \frac{1}{3-x}\,dx = \int \frac{1}{t}(-dx)$

$dt = -dx$

$-dt = dx$ $\qquad = -\ln(|t|) + k$

$= -\ln(|3-x|) + k$

(i) + (ii) : $\ln(|x-2|) - 2\cdot(-\ln(|3-x|) + k$

$= \ln(|x-2|) + 2\ln(|3-x|) + k$

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10) $\int \frac{x}{1-x^2}\,dx = \int \frac{1}{t}\,\frac{dt}{-2}$

$t = 1-x^2$ $\qquad = -\frac{1}{2}\ln(|t|) + k$

$dt = -2x\,dx$ $\qquad = -\frac{1}{2}\ln(|1-x^2|) + k$

$\frac{dt}{-2} = x\,dx$
